DO NOW

What do you know about Pascal's Triangle???

General Binomial Expansion:

$(x + \Delta x)^2 = x^2 + 2x\Delta x + (\Delta x)^2$

$(x + \Delta x)^3 = x^3 + 3x^2\Delta x + 3x(\Delta x)^2 + (\Delta x)^3$

$(x + \Delta x)^4 = x^4 + 4x^3\Delta x + 6x^2(\Delta x)^2 + 4x(\Delta x)^3 + (\Delta x)^4$

$(x + \Delta x)^n = \binom{n}{0}x^n + \binom{n}{1}x^{n-1}(\Delta x) + \binom{n}{2}x^{n-2}(\Delta x)^2 + \ldots \binom{n}{n}(\Delta x)^n$

$= x^n + nx^{n-1}\Delta x + \frac{n(n-1)}{2}x^{n-2}(\Delta x)^2 + \ldots (\Delta x)^n$

Pascal's Triangle:

* used for binomial expansion

1	← row 0
1 1	← row 1
1 2 1	← row 2
1 3 3 1	← row 3
1 4 6 4 1	← row 4
1 5 10 10 5 1	← row 5
⋮	

$(x+y)^5 = x^5 + 5x^4y + 10x^3y^2 + 10x^2y^3 + 5xy^4 + y^5$

3.2 Basic Differentiation Rules

THE CONSTANT RULE:

$\frac{d}{dx}[c] = 0$ where $c \in \mathbb{R}$

*If a function is a constant → graph is a horizontal line

∴ The derative (or slope) = 0

Examples: Find the derivative.

1. f(x) = 5
   $f'(x) = 0$
2. g(t) = -3
   $g'(t) = 0$
3. y = 12
   $y' = 0$

Find the derivative of $\frac{d}{dx}[x^n]$ using the limit process.

$\lim_{\Delta x \to 0} \frac{f(x+\Delta x) - f(x)}{\Delta x}$

$\lim_{\Delta x \to 0} \frac{(x+\Delta x)^n - x^n}{\Delta x}$

$\lim_{\Delta x \to 0} \frac{\cancel{x^n} + nx^{n-1}\Delta x + \frac{n(n-1)}{2}x^{n-2}(\Delta x)^2 \ldots (\Delta x)^n \cancel{- x^n}}{\Delta x}$

$\lim_{\Delta x \to 0} \frac{nx^{n-1}\Delta x + \frac{n(n-1)}{2}x^{n-2}(\Delta x)^2 \ldots + (\Delta x)^n}{\Delta x}$

$\lim_{\Delta x \to 0} nx^{n-1} + \frac{n(n-1)}{2}x^{n-2}\Delta x + \ldots + (\Delta x)^{n-1}$

$\boxed{\frac{d}{dx}[x^n] = nx^{n-1}}$

The Power Rule:

$\frac{d}{dx}[x^n] = nx^{n-1}$ where n is a rational #

Examples: Find the derivative.

4. $g(x) = x^7$
   $g'(x) = 7x^6$
5. $f(x) = x$
   $f'(x) = 1x^0$
   $f'(x) = 1$
6. $y = \frac{1}{x^5}$
   $y = x^{-5}$
   $y' = -5x^{-6}$
   $y' = \frac{-5}{x^6}$
7. $g(x) = \sqrt[3]{x}$
   $g(x) = x^{1/3}$
   $g'(x) = \frac{1}{3}x^{-2/3}$
   $g'(x) = \frac{1}{3x^{2/3}}$
   $g'(x) = \frac{1}{3\sqrt[3]{x^2}}$

The Constant Multiple Rule:

$\frac{d}{dx}[c(f(x))] = c(f'(x))$ where $c \in \mathbb{R}$

Examples: Find the derivative.

8. $\frac{d}{dx}[3x^2]$

$3 \cdot 2x$

$6x$

9. $g(t) = \frac{4t^2}{5}$

$g'(t) = \frac{4}{5} \cdot 2t$

$g'(t) = \frac{8t}{5}$

10. $y = \frac{5}{(2x)^3}$

$y = \frac{5}{8x^3}$

$y = \frac{5}{8} x^{-3}$

$y' = \frac{5}{8} \cdot -3x^{-4}$

$y' = \frac{-15}{8x^4}$

Sum and Difference Rule:

$\frac{d}{dx}[f(x) \pm g(x)] = f'(x) \pm g'(x)$

Prove the sum rule using the limit process.

$\lim_{\Delta x \to 0} \frac{f(x+\Delta x)+g(x+\Delta x) - (f(x)+g(x))}{\Delta x}$

$\lim_{\Delta x \to 0} \frac{f(x+\Delta x)+g(x+\Delta x) - f(x) - g(x)}{\Delta x}$

$\lim_{\Delta x \to 0} \frac{f(x+\Delta x) - f(x)}{\Delta x} + \frac{g(x+\Delta x) - g(x)}{\Delta x}$

$\lim_{\Delta x \to 0} \frac{f(x+\Delta x) - f(x)}{\Delta x} + \lim_{\Delta x \to 0} \frac{g(x+\Delta x) - g(x)}{\Delta x}$

$\boxed{f'(x) + g'(x)}$

Examples:

11. $f(x) = x^4 - 2x^3 + 5x^2 - 4x + 7$

$f'(x) = 4x^3 - 6x^2 + 10x - 4$

12. $g(x) = \frac{-x^4}{2} + 3x^2 - 3x$

$g(x) = -\frac{1}{2}x^4 + 3x^2 - 3x$

$g'(x) = -2x^3 + 6x - 3$

HOMEWORK

Worksheet 3.2